Homework 1 – Kinematic Relationships

1. The acceleration a of an object is given by the following expression:

$$a = \frac{d^2s}{dt^2}$$

where the symbols have the usual meanings.

(a) Starting with this expression for acceleration, show that, for an object moving with a constant acceleration, the velocity v of the object at time t is given by:

$$v = u + at$$

where u is the velocity at $t = 0$.

(b) A test vehicle moves in a straight line along a track. Its displacement, in metres, as a function of time is given by:

$$s = 24t - 2t^2$$

Determine:

(i) the time when the vehicle momentarily comes to rest;
(ii) the vehicle's displacement at this time;
(iii) the vehicle's acceleration.

(c) The test vehicle was timed as it travelled a measured distance of $(1{\cdot}000 \pm 0{\cdot}005)$ m along the track. The times recorded for this distance were:

1·21 s   1·23 s   1·24 s   1·20 s   1·22 s

Calculate:

(i) the average speed of the vehicle;
(ii) the absolute uncertainty in this speed.

2. Figure 1A shows a space shuttle shortly after take off.

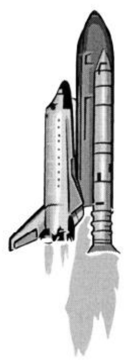

Figure 1A

Immediately after take-off, the vertical displacement of the shuttle for part of its journey can be described using the equation:

$$s = 3{\cdot}1t^2 + 4{\cdot}1t$$

(a) Determine, by differentiation, the equation for the vertical velocity of the shuttle.

(b) Calculate the time at which the vertical velocity will be 72 ms^{-1}.

(c) Calculate the vertical linear acceleration at this time.

3. A stunt driver is attempting to "loop the loop" in a car as shown in Figure 1.
Before entering the loop, the car accelerates along a horizontal track.

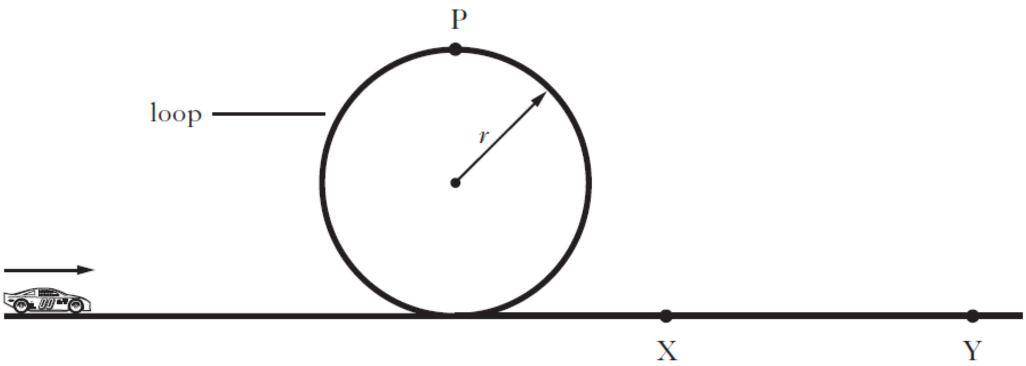

Figure 1

When the car exits the loop, the driver starts breaking at point X. For one particular run, the displacement of the car from point X **until the car comes to rest** at point Y is given by the equation:

$$s = 9{\cdot}1t - 3{\cdot}2t^2$$

Sketch a graph to show how the displacement of the car varies with time between points X and Y.
Numerical values are required on both axes.

4.

A car on a long straight track accelerates from rest. The car's run begins at time $t = 0$. Its velocity v at time t is given by the equation:

$$v = 0{\cdot}135t^2 + 1{\cdot}26t$$

where v is measured in ms^{-1} and t is measured in s.

Using **calculus** methods:

(a) determine the acceleration of the car at t = 15·0 s;

(b) determine the displacement of the car from its original position at this time.