Homework 3 – Rotational Dynamics

1. A gyroscope is a spinning wheel or disc in which the axis of rotation is free to assume any orientation by itself.

   (a) A mass m rotates at a distance r from a fixed axis.
   State the expression for its moment of inertia.

   (b) A toy gyroscope consists of an axle, a narrow ring and spokes as shown in Figure 1A.

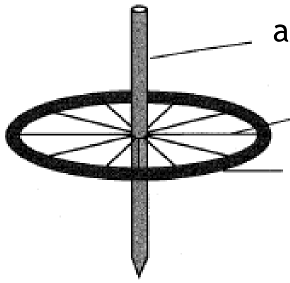


Figure 1A

   The mass of the axle and spokes is negligible compared to the mass of the narrow ring.
   The narrow ring has a mass of 1·5 kg and an average radius of 0·20 m.
   Show that the moment of inertia of the gyroscope is 0·060 kg m^2.

   (c) The axle of the gyroscope has a radius of 4·0 mm.
   The gyroscope is made to spin using a thin cord. A 0·50 m length of thin cord is wound round the axle, as shown in Figure 1B.

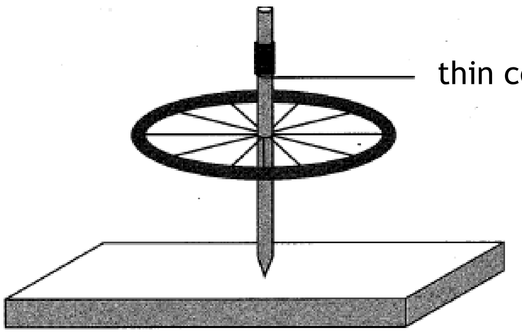


Figure 1B

   The cord is pulled with a steady horizontal force of 25 N.
   A constant frictional torque of 0·070 Nm opposes the motion of the gyroscope.

   (i) Calculate the resultant torque acting on the gyroscope.
   (ii) Calculate the angular acceleration of the gyroscope.
   (iii) Show that the angular displacement of the gyroscope is 125 radians just as the cord fully unwinds.
   (iv) Calculate the maximum angular velocity of the gyroscope.
   (v) After the cord is fully unwound, the frictional torque remains constant.
   Calculate how long it takes for the angular velocity of the gyroscope to decrease to 4·2 rad s^{-1}.

2. A student uses two methods to calculate the moment of inertia of a solid cylinder about its central axis.

(a) In the first method, the student measures the mass of the cylinder to be 0·115 kg and the diameter to be 0·030 m.

Calculate the moment of inertia of the cylinder.

(b) In a second method, the student allows the cylinder to roll down a slope and measures the final speed at the bottom of the slope to be 1·60 ms^{-1}.

The cylinder has a diameter of 0·030 m and the slope has a height of 0·25 m, as shown in Figure 2.

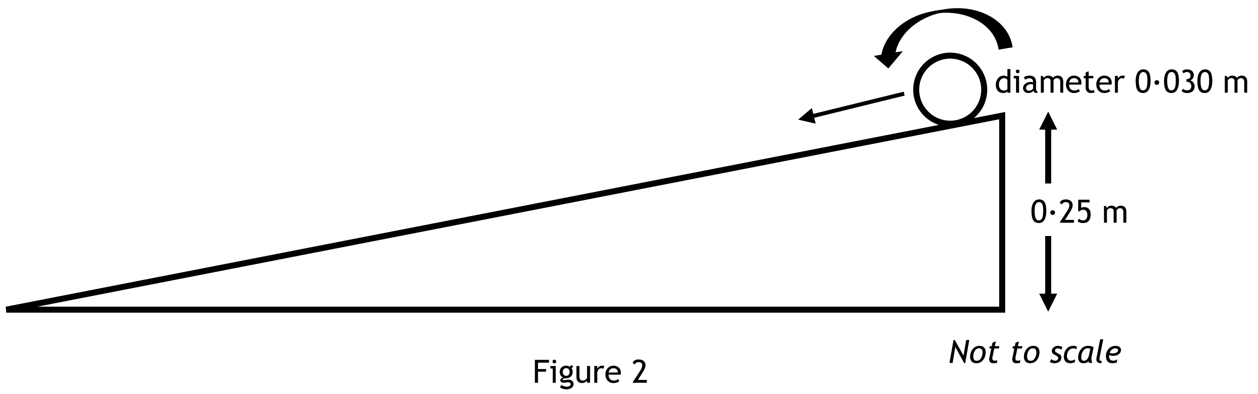


Figure 2

Using the conservation of energy, calculate the moment of inertia.

(c) Explain why the moment of inertia found in part (b) is greater than in part (a).

3. A playground roundabout has a radius of 2·0 m and a moment of inertia of 500 kg m^2 about its axis of rotation.

A child of mass 25 kg runs tangentially to the stationary roundabout and jumps onto its outer edge, as shown in Figure 3.

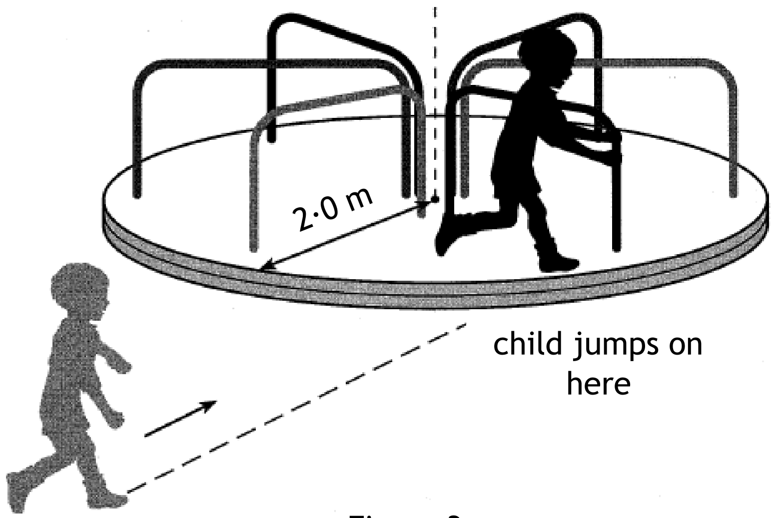


Figure 3

(a) Show that, with the child at the outer edge, the combined moment of inertia of the roundabout and the child is 600 kg m^2.

(b) State what is meant by *conservation of angular momentum*.

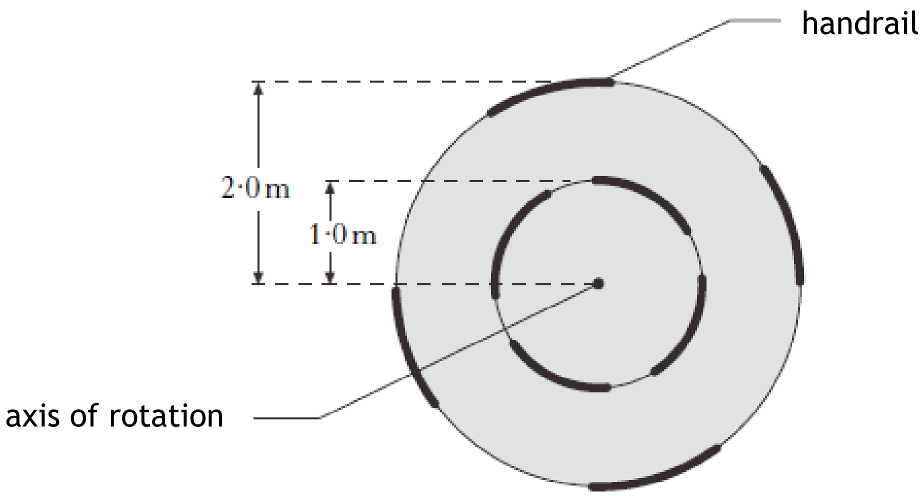
(c) At the point of jumping onto the roundabout, the tangential speed of the child is 2·4 ms^{-1}.
At **this point**, calculate:
(i) the linear momentum of the child;
(ii) the angular momentum of the child about the axis of rotation of the roundabout.

(d) Calculate the angular velocity of the roundabout and the child just after the child jumps on.

(e) Calculate the loss of kinetic energy as the child jumps onto the roundabout.

(f) The roundabout with the child on-board makes half a complete revolution before coming to rest.
Calculate the frictional torque acting on the roundabout.

4. A playground roundabout consists of a uniform disc and eight handles in the form of arcs as shown in Figure 4A. The uniform disc has a radius of 2·0 m and a moment of inertia, about the axis of rotation, of 500 kg m^2. The handrails each have a mass of 15 kg. The inner handrails are 1·0 m from the centre.



roundabout viewed from above

Figure 4A

(a) Calculate the moment of inertia of the roundabout about the axis of rotation.

(b) A boy of mass 40 kg stands in a vertical position on the rim of the roundabout as shown in Figure 4B. The roundabout is rotating about its axis of rotation with a constant angular velocity. The roundabout takes 3·0 s to make one complete revolution. The effect of friction at the axis of rotation can be neglected.

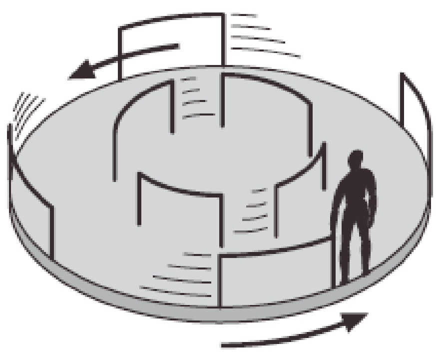


Figure 4B

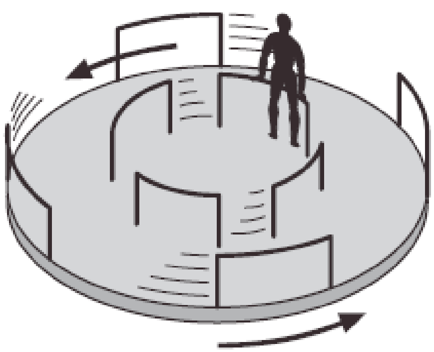


Figure 4C

Calculate the angular velocity, in radians per second, of the roundabout.

(c) The boy now moves to a position 1·0 m from the axis of rotation, as shown in Figure 4C. He holds the handrails to steady himself and maintain his body in a vertical position.

(i) Explain why the angular velocity of the roundabout changes. Assume that the boy's mass does not alter and that no external torque is applied.

(ii) Hence, or otherwise, calculate the new angular velocity of the roundabout after the boy moves inwards.

(d) (i) Show by calculation that the kinetic energy of the system has not been conserved.
(ii) Account for this change in kinetic energy.