

## Homework 4 - Gravitation

1. (a) The Moon orbits the Earth due to the gravitational force between them.
  - (i) Calculate the magnitude of the gravitational force between the Earth and the Moon.
  - (ii) Hence, calculate the tangential speed of the Moon in orbit around the Earth.
  - (iii) Define the term *gravitational potential* at a point in space.
  - (iv) Calculate the potential energy of the Moon in its orbit.
  - (v) Hence, calculate the total energy of the Moon in its orbit.
  
- (b) (i) Derive an expression for the escape velocity from the surface of an astronomical body.
  - (ii) Calculate the escape velocity from the surface of the Moon.
  
2. (a) (i) State what is meant by *gravitational field strength*.
  - (ii) The gravitational field strength at the surface of Mars is  $3.7 \text{ N kg}^{-1}$ .  
The radius of Mars is  $3.4 \times 10^3 \text{ km}$ .  
(A) Use Newton's Universal Law of Gravitation to show that the mass of Mars is given by:
$$M = \frac{gr^2}{G}$$
where the symbols have their usual meanings.  
(B) Calculate the mass of Mars.
  
- (b) A spacecraft of mass  $100 \text{ kg}$  is in circular orbit  $300 \text{ km}$  above the surface of Mars.
  - (i) Show that the force exerted by Mars on the spacecraft is  $3.1 \times 10^2 \text{ N}$ .
  - (ii) Calculate the period of the spacecraft's orbit.

3. An article in a newspaper describing the launch of a space rocket states:

*“The shuttle blasts off, then comes the tremendous pressure of three G’s and the sudden release into weightlessness as the ship leaves the gravitational field behind.”*

Use your knowledge of physics to comment on this statement.

4. (a) Show that the gravitational field strength at the surface of Pluto, mass  $M_P$ , is given by:

$$g = G \frac{M_P}{r^2}$$

where the symbols have their usual meanings.

(b) Figure 4A shows how the gravitational potential varies with distance from the centre of Pluto.

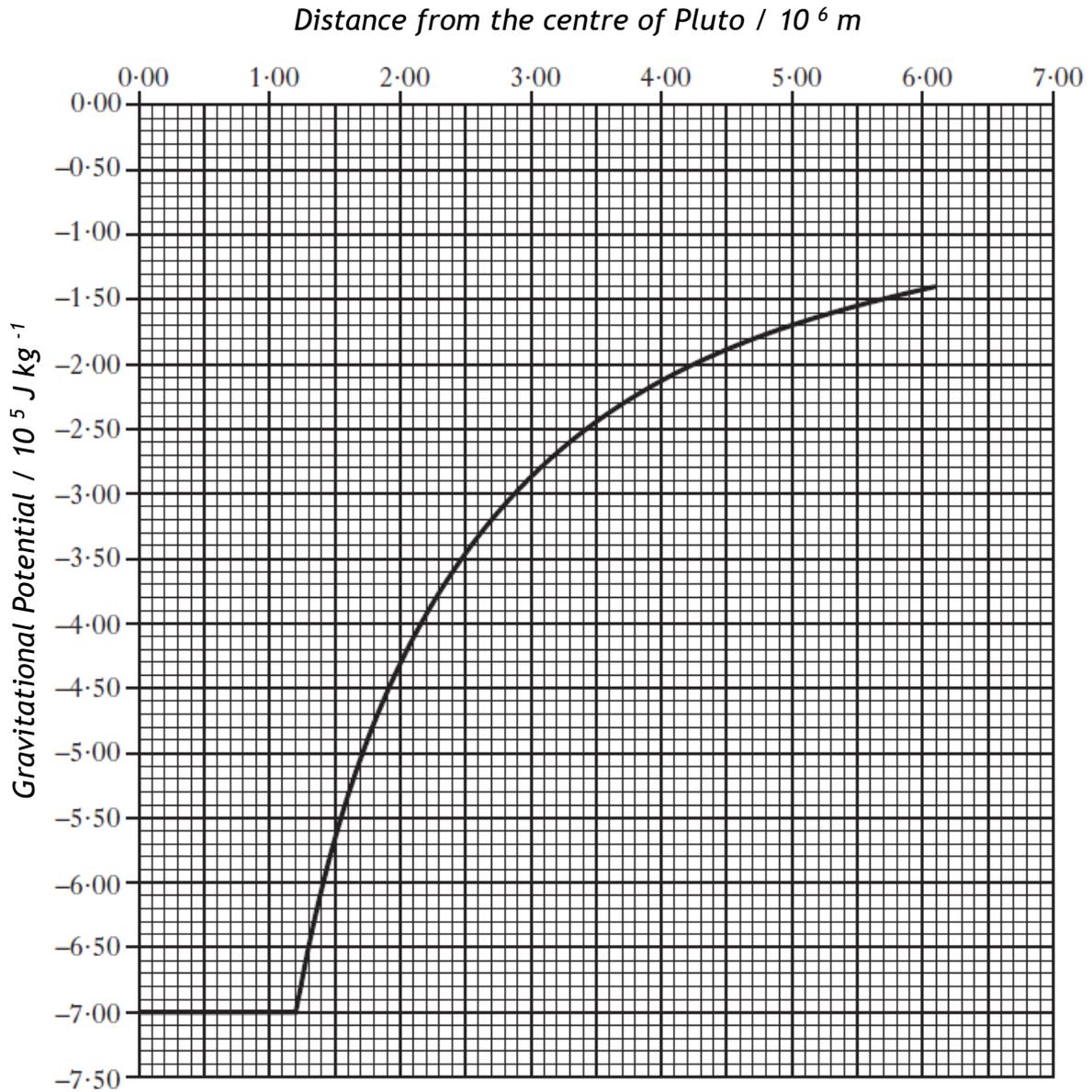


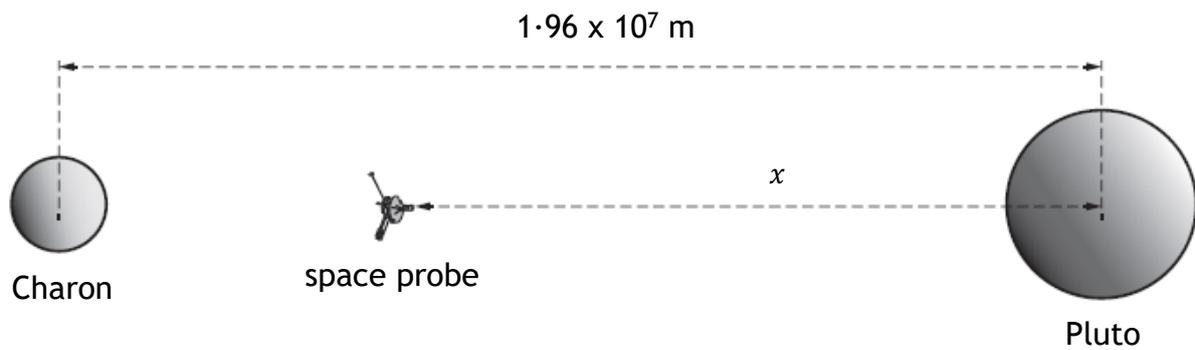
Figure 4A

(i) The mass of Pluto is  $1.27 \times 10^{22}$  kg.

Calculate the gravitational field strength at the surface of Pluto.

- (ii) A meteorite strikes the surface of Pluto and ejects a lump of ice of mass 112 kg. The ice is captured in an orbit  $1.80 \times 10^6$  m from the centre of Pluto. Calculate the gravitational potential energy of the ice at this height.

- (c) In 2015, the New Horizons space probe arrived at Pluto. It moved between Pluto and its Moon, Charon, as shown in Figure 4B. Pluto has a mass seven times that of Charon and their average separation is  $1.96 \times 10^7$  m.



Calculate the distance,  $x$ , from the centre of Pluto where the resultant gravitational force acting on the probe is zero.

Ignore any orbital motion of the two objects.

5. The gravitational force exerted by the Earth on a satellite causes it to maintain a circular orbit of radius  $r$ .

(a) By equating the expressions for gravitational force and centripetal force, show that:

$$r^3 = \frac{GM_E}{4\pi^2} T^2$$

where the symbols have their usual meanings.

(b) The orbital period of a geostationary satellite is equal to the period of rotation of the Earth about its axis.

Calculate:

- (i) the height of the satellite above the Earth's surface;
- (ii) the speed of the satellite in its orbit.

(c) Another satellite is in an orbit of  $6.7 \times 10^6$  m around the Earth. This satellite is to be boosted to escape velocity.

- (i) State what is meant by the term *escape velocity*.
- (ii) Calculate the escape velocity of the Earth.