Homework 6 – Stellar Physics

1. Cygnus X-1 is an x-ray source in the constellation Cygnus that astrophysicists believe is a black hole. An artist's impression is shown in Figure 1A.

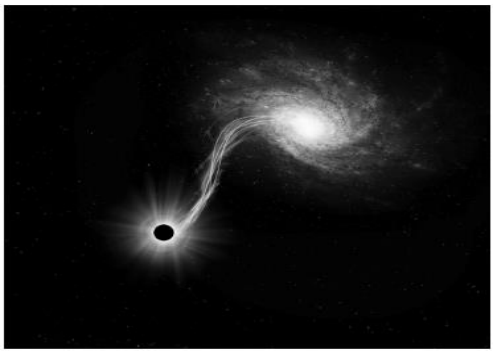

Figure 1A

The mass of the black hole has been determined to be 14·8 solar masses.

(a) (i) State what is meant by the Schwarzschild radius of a black hole.
(ii) Calculate the Schwarzchild radius of the black hole in Cygnus X-1.

(b) The Hertzsprung-Russell (H-R) diagram shown in Figure 1B shows the relationship between luminosity and surface temperature of stars.

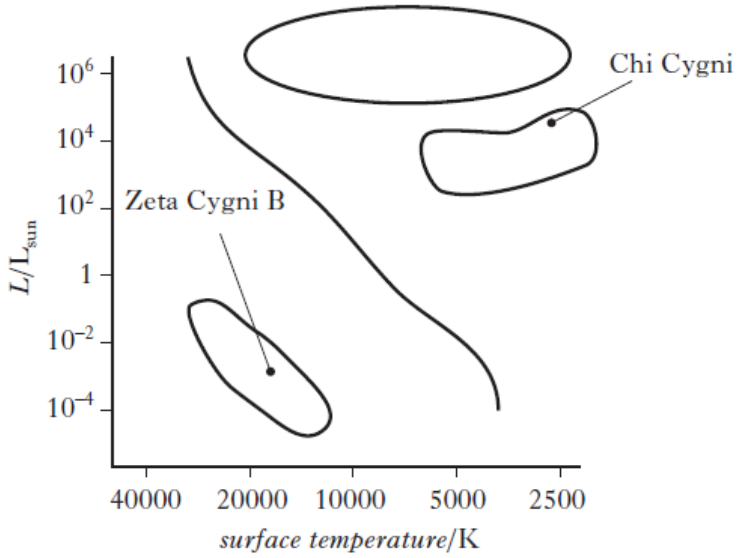

Figure 1B

Zeta Cygni B and Chi Cygni are two stars in the constellation Cygnus.
They are shown on the H-R diagram. Chi Cygni is more luminous than Zeta Cygni B.
Describe two other differences between these stars.

(c) Another star, Aldebaran B, is a distance of $6{\cdot}16 \times 10^{17}$ m from the Earth.
The luminosity of Aldebaran B is $2{\cdot}32 \times 10^{25}$ W and its temperature is determined to be $3{\cdot}4 \times 10^{3}$ K.
(i) Calculate the radius of Aldebaran B.
(ii) Calculate the apparent brightness of Aldebaran B as observed from Earth.

2. Hertzsprung-Russell (H-R) diagrams are widely used by physicists and astronomers to categorise stars. Figure 2A shows a simplified H-R diagram.

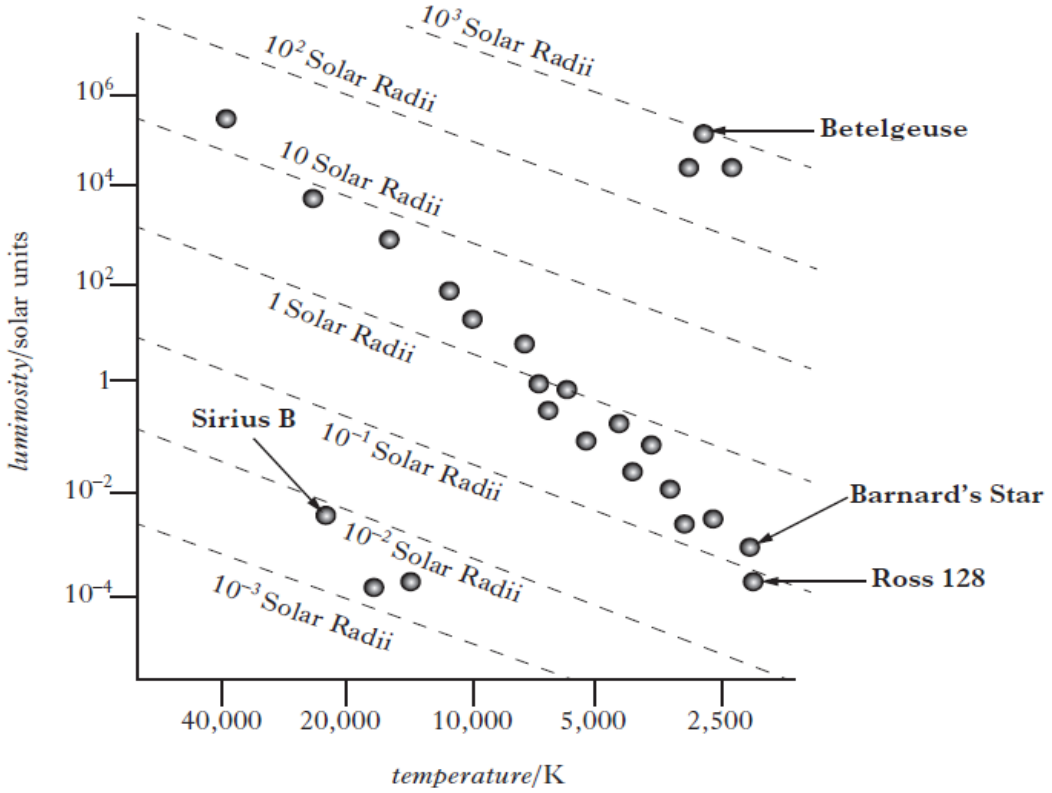

Figure 2A

(a) State what class of star Sirius B is.

(b) Estimate the radius of Betelgeuse.

(c) Ross 128 and Barnard's Star have a similar temperature but Barnard's Star has a slightly greater luminosity.
Determine what other information this tells you about the two stars.

(d) During the life cycle of the Sun, its position in the H-R diagram is expected to change as shown by the arrowed line in Figure 2B.

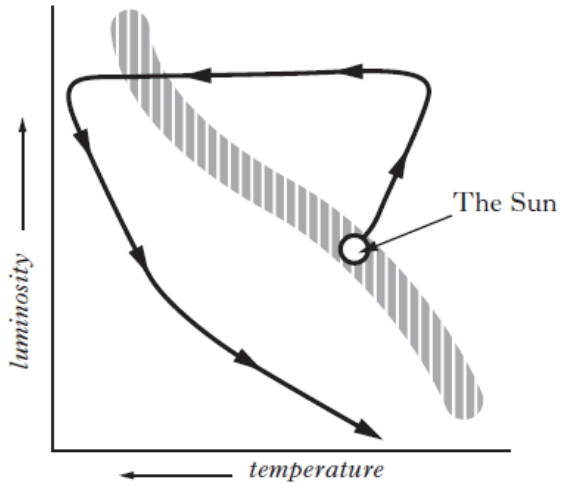

Figure 2B

Describe the changes that occur to the Sun during its expected life cycle.

(e) Hydrogen fusion in a star is a result of a proton-proton chain. The process eventually results in the production of a helium-4 nucleus.

(i) Show that the percentage loss of mass from four protons to one helium-4 nucleus is 0·7%.

(ii) The luminosity of the Sun is $3{\cdot}8 \times 10^{26}$ W. Using Einstein's energy equation, show that the mass of hydrogen lost per second is $4{\cdot}2 \times 10^{9}$ kg.

(iii) Estimate the lifetime of the Sun in seconds. Assume the mass of hydrogen in the Sun to be the same as the mass of the Sun.

(f) The "no greenhouse" temperature of a planet is the average surface temperature of a planet in the absence of any greenhouse effect. The "no greenhouse" temperature of a planet, in kelvin, is calculated by:

$$T = 280\left(\frac{1 - reflectivity}{d^2}\right)^{\frac{1}{4}}$$

where d is the distance from the Sun in astronomical units (AU).
The reflectivity is a measure of the percentage of energy reflected from the surface; 1 represents 100% reflectivity and 0 represents no reflectivity.

Mercury has a reflectivity of 0·12 and is $5{\cdot}8 \times 10^{10}$ m from the Sun.
Calculate its "no greenhouse" temperature.

3. Information about two stars is given in the table.

	Star A	**Star B**
Radius (m)	$2{\cdot}0 \times 10^{9}$	$8{\cdot}0 \times 10^{9}$
Surface Temperature (K)	7000	3500
Distance from Earth (ly)	20	20

(a) State which star, if any, appears brighter in the Earth's night sky.
Justify your answer by calculation.

(b) Star A could collapse to form a black hole.
Calculate the Schwarzschild radius of this black hole, assuming the density of Star A is $2{\cdot}5 \times 10^{3}$ kg m^{-3}.

Note: The density, ρ, of a mass, m, with volume, V, is calculated using $\rho = \frac{m}{V}$.

4. (a) All stars on the main sequence release energy by converting hydrogen to helium.
This process is known as the proton-proton (p-p) chain.
One stage in the p-p chain is shown.

$$ {}_1^1H + {}_1^1H \rightarrow {}_1^2H + x + y $$

Name the particles x and y.

(b) The expression

$$ \frac{L}{L_0} = 1 \cdot 5 \left(\frac{M}{M_0}\right)^{3 \cdot 5} $$

can be used to approximate the relationship between a star's mass M and its luminosity L.

L_0 is the luminosity of the Sun (1 solar unit) and M_0 is the mass of the Sun.
This expression is valid for stars of mass between $2M_0$ and $20M_0$.
Spica is a star which has mass $10 \cdot 3M_0$.
Determine the approximate luminosity of Spica in solar units.