Grove Academy

Advanced Higher Physics

Rotational Motion and Astrophysics Problem Booklet

EXERCISE 1.1

Equations of motion

1. The displacement, s in metres, of an object after a time, t in seconds, is given by

$$s = 90t - 4t^2$$

   (a) Determine, by differentiation, the equation for its velocity.
   (b) Determine the time at which the velocity will be zero.
   (c) Show that the acceleration is a constant and state its value.

2. Given that $a = \frac{dv}{dt}$ show by integration that the velocity, v, is given by $v = u + at$.
   State clearly the meaning of the symbol, u, in this equation.

3. Given that $v = \frac{ds}{dt}$ and $v = u + at$, show by integration that $s = ut + ½ at^2$, where the symbols have their usual meaning.

4. The displacement, s, of a moving object after a time, t, is given by $s = 8 - 10t + t^2$.
   Show that the unbalanced force acting on the object is constant.

5. The displacement, s, of an object after time, t, is given by $s = 3t^3 + 5t$.
   (a) Derive an expression for the acceleration of the object.
   (b) Explain why this expression indicates that the acceleration is not constant.

6. A trolley is released from the top of a runway which is 6 m long.
   The displacement s in metres, of the trolley is given by the expression $s = 5t + t^2$ where t is in seconds. Determine:
   (a) an expression for the velocity of the trolley
   (b) the acceleration of the trolley
   (c) the time it takes the trolley to reach the bottom of the runway
   (d) the velocity of the trolley at the bottom of the runway.

7. A box slides down a smooth slope with an acceleration of 4 m s^{-2}. The velocity of the box at a time t = 0 is 3 m s^{-1} down the slope.
   Using $a = \frac{dv}{dt}$ show by integration that the velocity, v, of the box is given by $v = 3 + 4t$.

8. The equation for the velocity, v, of a moving trolley is $v = 2 + 6t$.
   Using $v = \frac{ds}{dt}$ derive an expression for the displacement, s, of the trolley.

9. A projectile is launched from the top of a building with an initial speed of 20 ms^{-1} at an angle of 30° to the horizontal. The height of the building is 30 m.
   (a) Calculate how long it takes the projectile to reach the ground.
   (b) Calculate the velocity of the projectile on impact with the ground, (magnitude and direction).

EXERCISE 2.1

Angular motion

1. Convert the following from degrees to radians:

30^o 45^o 60^o 90^o
180^o 270^o 360^o 720^o

2. Convert the following from radians to degrees:

1 rad 10 rad 0.1 rad π rad 2π rad $\frac{\pi}{2}$ rad $\frac{\pi}{6}$ rad.

3. Convert the following from revolutions per minute to radians per second:

33 rpm 45 rpm 78 rpm 300 rpm.

4. Using calculus notation, state the expression for:
   (a) the angular velocity in terms of the angular displacement
   (b) the angular acceleration in terms of the angular velocity
   (c) the angular acceleration in terms of the angular displacement.

5. State the three equations which can be used when an object moves with a constant angular acceleration, α.
   State the meaning of each symbol used.

6. A disc is slowed uniformly at 5.0 rads^{-2} for 4.0 s. The initial angular velocity is 200 rads^{-1}.
   (a) Calculate the angular velocity at the end of the four seconds.
   (b) Calculate the angular displacement in this time.

7. The angular velocity of an engine is increased from 800 rpm to 3 000 rpm in 8.0 s.
   (a) Determine the angular acceleration. You may assume this is uniform.
   (b) Calculate the total angular displacement.
   (c) Calculate how many revolutions the engine makes during this 8.0 s.

8. A wheel accelerates uniformly from rest at 3.0 rad s^{-2} for 5.0 s.
   (a) Calculate:
      (i) the final angular velocity after 5.0 s
      (ii) the angular displacement after 5.0 s.
   (b) The wheel has a radius of 1.50 m.
   Calculate the linear velocity at a point on its rim at the end of the 5.0 s.

9. **Radius of Earth = 6.4×10^3 km**
   Radius of Earth's orbit = 1.5×10^8 km
   Period of Earth about Sun = 365 days
   Geostationary orbit radius = 3.6×10^4 km
   Radius of Moon's orbit = 3.8×10^5 km
   Period of Moon about Earth = 28 days

   (a) Calculate the angular velocity in rad s^{-1} of
      (i) the Earth about the sun
      (ii) the Moon about the Earth
      (iii) an object on the Earth's surface about its axis of rotation
      (iv) a geostationary satellite.
   (b) Calculate the tangential velocity in ms^{-1} of each of the above quantities in part (a).

10. Derive the expression $v = r\omega$ for a particle in circular motion.

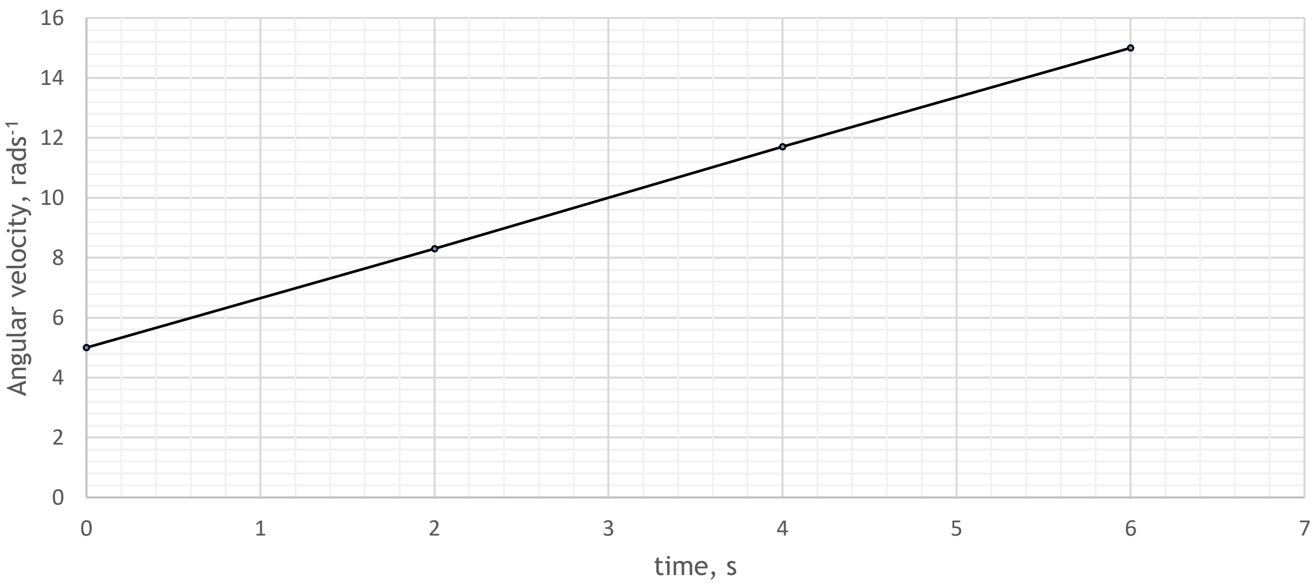
EXERCISE 2.2

Angular Motion

1. If 2π radians equals 360°, calculate the number of degrees in one radian.

2. Calculate the angular velocity in rads^{-1} of the second hand of an analogue watch.

3. The graph below shows the variation of angular velocity with time for a rotating body.



(a) Calculate the angular displacement θ covered in the first 3 seconds.
(b) Calculate the total angular displacement for the 6 seconds.
(c) Calculate the angular acceleration of the rotating body.

4. A wheel accelerates uniformly from rest. After 12 s the wheel is completing 100 revolutions per minute (r.p.m.)
(a) Convert 100 r.p.m. to its equivalent value in rads^{-1}.
(b) Calculate the average angular acceleration of the wheel.

5. The angular velocity of a car engine's drive shaft is increased from 100 rad s^{-1} to 300 rad s^{-1} in 10 s.
(a) Calculate the angular acceleration of the drive shaft.
(b) Calculate the angular displacement during this time.
(c) A point on the rim of the drive shaft is at a radius of 0.12 m.
Calculate the distance covered by this point in the 10 s time interval.

6. Use calculus methods to derive the equations for angular motion. The method is very similar to that for linear motion.

Note: in the unit or course assessment you may be asked to derive the linear motion equations but **not** the angular motion equations.

EXERCISE 3.1

Central force

1.
   (a) State the equation between radial acceleration and angular velocity.
   (b) State the units of angular and radial acceleration.
   (c) Explain the difference between angular and radial acceleration.

2. Derive the expression $a = r\omega^2$ for the radial acceleration of an object.

3. The central force maintaining an object in a circular orbit is given by $F = mr\omega^2$.
   Sketch graphs showing the variations of:
   (a) central force with mass of the object
   (b) central force with radius of the object
   (c) central force with angular velocity of the object.

4. A piece of string has a breaking force of 56 N.
   This string is used to whirl a mass of 150 g in a horizontal circle.
   (a) The 150 g mass moves in a horizontal circle of radius 1.2 m.
   Calculate the maximum angular velocity of the mass.
   (b) The mass is rotated at 85 rpm.
   Calculate the maximum possible radius of the circular orbit.

5. A swing ball, on a cord of length 1.5 m, has a mass of 2.0 kg. After being hit by a bat, the ball moves in a horizontal circle of radius 0.50 m with a steady speed of 1.3 m s^{-1}.
   (a) Sketch the path of the ball on the string.
   (b) Calculate the central acceleration of the ball.
   (c) Draw a free-body diagram showing all the forces on the ball while moving in a horizontal circle.
   Use your diagram to determine the tension in the string.

6. A 3.0 kg mass is whirled in a vertical circle of radius 0.75 m at a steady speed of 8.0 m s^{-1}.
   (a) Calculate the tension in the string at the top of the circle.
   (b) Calculate the tension in the string at the bottom of the circle.

7. A hump-backed bridge is in the form of a circular arc of radius 35 m.
   Calculate the greatest speed with which a car can cross the bridge without leaving the ground at its highest point.

8. In a space flight simulator an astronaut is rotated at 20 rpm in a pod which is at the end of an arm of radius 5.0 m.
   (a) Show that the central force on the astronaut is 2.2g (2.2 times greater than the gravitational force on Earth).
   (b) Calculate the rotation rate that would give a ‘simulated’ gravity of 3g.

9. Comment on the words centripetal and centrifugal with respect to angular motion.

EXERCISE 3.2

Circular Motion

1. An Earth satellite is required to be in a circular orbit at a distance of 7.5 x 10^6 m from the centre of the Earth. The central force is due to the gravitational force. The acceleration due to the Earth's gravity at this point is 7.0 ms^{-2}.

   Calculate:
   (a) the required satellite speed
   (b) the period of revolution of the satellite.

2. What would be the period of rotation of the Earth about its axis if its speed of rotation increased to such an extent that an object at the equator became 'weightless'?

   (Hint: $mg = \frac{mv^2}{r}$)

3. A sphere of mass 0.20 kg is rotating in a circular path at the end of a string 0.80 m long. The other end of the string is fixed. The period of the motion is 0.25 s.
   (a) Calculate the tension in the string, which you may assume to be horizontal.
   (b) In practice the string is not horizontal. Explain why this is so.
   (c) Draw a force diagram for the sphere.
   (d) From this calculate the angle the string would make with the horizontal.

4. The moon takes 27.3 days (2.0 x 10^6 s) to complete one orbit of the Earth.
   The distance between the centres of the Earth and Moon is 4.0 x 10^8 m.
   Calculate the magnitude of the Moon's acceleration towards the Earth.

5. A ball of mass 2.0 kg is attached to a string 1.6 m long and is made to travel in a vertical circle. The ball passes its highest point with a speed of 5.0 ms^{-1}.
   (a) Calculate the kinetic energy of the ball at its highest point.
   (b) Calculate its potential energy when it is at the highest point (with reference to its lowest point).
   (c) Calculate its kinetic energy at the lowest point.
   (d) Calculate its speed at the lowest point.
   (e) Calculate the tension in the string at the highest and lowest points.
   (f) Calculate the **smallest** speed the ball could have at the highest point in order to be able to complete a vertical circle at all.

6. An old humpback bridge has a radius of curvature of 20 m. Calculate the maximum speed at which a car can pass over this bridge if the car is not to leave the road surface

7. A pail of water is swinging in a vertical circle of radius 1.2 m, so that the water does not fall out. Calculate the minimum linear speed required for the pail of water.
   Convert this speed into an angular velocity.

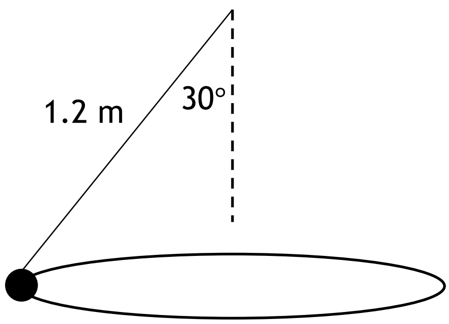
8. An object of mass 0.20 kg is connected by a string to an object of half its mass. The smaller mass is rotating at a radius of 0.15 m on a table which has a frictionless surface. The larger mass is suspended through a hole in the middle of the table.
   Calculate the number of revolutions per minute the smaller mass must make so that the larger mass is stationary.

Banking of a Track

9. A circular track of radius 60 m is banked at angle θ.
   A car is driven round the track at 20 m s^{-1}.
   (a) Draw a free-body diagram showing the forces acting on the car.
   (b) Calculate the angle of banking required so that the car can travel round the track without relying on frictional forces (i.e. no side thrust supplied by friction on the track surface).

Conical Pendulum

10. A small object of mass m revolves in a horizontal circle at constant speed at the end of a string of length 1.2 m. As the object revolves, the string sweeps out the surface of a right circular cone.



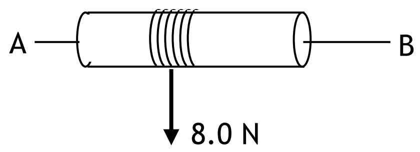
The cone has semi-angle 30°.
Calculate:
(a) the speed of the object
(b) the period of the motion.

[Hint: try resolving the tension in the string into horizontal and vertical components.]

EXERCISE 4.1

Torque and moment of inertia

1.
   (a) State what is meant by the moment of a force.
   (b) Give two examples illustrating the moment of a force.
2.
   (a) State the equation relating torque and tangential force.
   (b) State the equation relating torque and angular acceleration.
3. The moment of inertia of an object depends on two quantities.
   State clearly these two quantities.
4. The moment of inertia of a rod about an axis through its centre is different to the moment of inertia of the same rod about an axis through one end.
   Explain why this is so and justify which arrangement has the larger moment of inertia.
5. A wheel has very light spokes. The mass of the wheel and tyre is 2.0 kg and the radius of the wheel is 0.80 m.
   Calculate the moment of inertia of the wheel. State any assumptions that you have made.
6. A cylindrical solid drum is free to rotate about an axis AB as shown below.



   The radius of the drum is 0.30 m. The moment of inertia of the drum about AB is 0.40 kg m^2.
   A rope of length 5.0 m is wound around the drum and pulled with a constant force of 8.0 N.
   (a) Calculate the torque on the drum.
   (b) Determine the angular acceleration of the drum.
   (c) Calculate the angular velocity of the drum just as the rope leaves the drum. You may assume that the drum starts from rest.
7. A hoop has a radius of 0.20 m and a mass of 0.25 kg.
   (a) Calculate the moment of inertia of the hoop.
   (b) Calculate the torque required to give the hoop an acceleration of 5.0 rad s^{-2}.
8. A sphere has a moment of inertia of $\frac{2}{5}MR^2$ where M is the total mass of the sphere and R is the radius.
   (a) Calculate the moment of inertia of the Earth as it spins on its axis.
   State any assumptions made.
   (b) Calculate the tangential speed at the surface of the Earth at the Equator.
9. Two children are playing on a roundabout. One child, Anne, of mass 50 kg, stands on the roundabout 1.25 m from the axis of rotation. The other child, Robert, starts the roundabout by applying a constant torque of 200 Nm at the rim for 3 s.
   When rotating there is a constant frictional torque of 25 N m acting on the roundabout.
   Robert stops pushing and the roundabout comes to rest.
   The moment of inertia of the roundabout alone is 500 kg m^2.
   (a) Calculate the maximum angular velocity of the roundabout.
   (b) Find the time taken for the roundabout to come to rest.

EXERCISE 5.1

Angular momentum and rotational kinetic energy

1. State the law of conservation of angular momentum.

2. State the expression for the angular momentum of an object in terms of its moment of inertia.

3. State the equation for the rotational kinetic energy of a rigid object.

4. A bicycle wheel has a moment of inertia of 0.25 kg m^2 about its hub.
   Calculate the angular momentum of the wheel when rotating at 120 r.p.m.

5. A model aeroplane is flying in a horizontal circle at the end of a light wire. The mass of the aeroplane is 2.0 kg. The radius of the circular path is 20 m. The aeroplane makes 40 revolutions in one minute.
   (a) Calculate the linear velocity of the aeroplane.
   (b) Calculate the angular momentum of the aeroplane about the centre of the circle.
   (c) The wire suddenly breaks. State the new angular momentum of the aeroplane about the centre of the circle. Justify your answer.

6. A shaft has a moment of inertia of 20 kg m^2 about its central axis.
   The shaft is rotating at 10 rpm. This shaft is locked onto another shaft, which is initially stationary. The second shaft has a moment of inertia of 30 kg m^2.
   (a) Calculate the angular momentum of the combination after the shafts are locked together.
   (b) Calculate the angular velocity of the combination after the shafts are locked together?

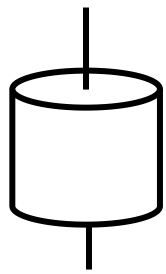
7. Identify which of the following are vector quantities?

torque	**moment of inertia**
angular velocity	**tangential force angular acceleration**
rotational kinetic energy	**radius of a circular motion**

8. Two children are playing on a roundabout of mass 250 kg. The roundabout can be considered to be a solid disc of diameter 3.0 m. $\left(I_{disc} = \frac{1}{2}mr^2\right)$
   (a) One child of mass 40 kg stands on the rim of the roundabout.
   The other child of mass 60 kg is positioned half way between the rim and the centre.
   Calculate the total moment of inertia of the roundabout and children.
   (b) Calculate the rotational kinetic energy of this system when it is rotating at 35 rpm.

9. A disc has a moment of inertia of 2.5 kgm^2. The disc is rotating at 2.0 $rads^{-1}$.
   (a) Calculate the kinetic energy of the disc.
   (b) Calculate how much energy needs to be supplied to increase its angular velocity to 15 $rads^{-1}$.

10. A solid cylinder and a hollow cylinder have the same mass and the same radius.



(a) State which cylinder has the larger moment of inertia about the central axis as shown opposite.
You must justify your answer.

(b) The cylinders do not have the same length.
State how this affects your answer to part (a)?
You must justify your answer.

11. A cylinder of mass 3.0 kg rolls down a slope without slipping. The radius R of the cylinder is 50 mm and its moment of inertia is ½MR^2. The slope has a length of 0.30 m and is inclined at 40º to the horizontal.

(a) Calculate the loss in gravitational potential energy as the cylinder rolls from the top of the slope to the bottom of the slope.

(b) Calculate the speed with which the cylinder reaches the bottom of the slope.

12. A turntable is rotating freely at 40 rpm about a vertical axis. A small mass of 50 g falls vertically onto the turntable and lands at a distance of 80 mm from the central axis. The rotation of the turntable is reduced to 33 rpm.
Calculate the moment of inertia of the turntable.

13. A CD of mass 0.020 kg and diameter 120 mm is dropped onto the centre of a rotating turntable. The turntable has a moment of inertia about its axis of rotation of 5.0×10^{-3} kg m^2.
The turntable was initially rotating at 3.0 rads^{-1}.
Determine the common angular velocity of the turntable and the CD.

14. A skater with her arms pulled in has a moment of inertia of 1.5 kg m^2 about a vertical axis through the centre of her body. With her arms outstretched the moment of inertia is increased to 10 kg m^2.
With her arms pulled in, the skater is spinning at 30 rad s^{-1}. The skater then extends her arms.

(a) Calculate her final angular speed.

(b) Calculate the change in kinetic energy.

(c) Explain why there is a change in kinetic energy.

15. A skater is spinning at 3.0 rad s^{-1} with her arms and one leg outstretched.

(a) The angular speed is increased to 25 rad s^{-1} when she draws her arms and leg in.
Explain why this movement of her arms and leg affects the rotational speed.

(b) Her moment of inertia about her spin axis is 5.0 kg m^2 with her arms and leg outstretched. Calculate her moment of inertia when her arms and leg are drawn in.

16. A roundabout has a moment of inertia of 300 kg m^2 about its axis of rotation. Three children, each of mass 20 kg, stand 2.0 m from the centre of the stationary roundabout. They all start to run round the roundabout in the same direction until they reach a speed of 3.0 m s^{-1} relative to the roundabout.
Calculate the angular velocity of the roundabout.

17. A disc is rotating at 100 rpm in a horizontal plane about a vertical axis. A small piece of plasticine is dropped vertically onto the disc and sticks at a position 50 mm from the centre of the disc. The plasticine has a mass of 20 g. The disc is slowed to 75 rpm.
Calculate the moment of inertia of the disc.

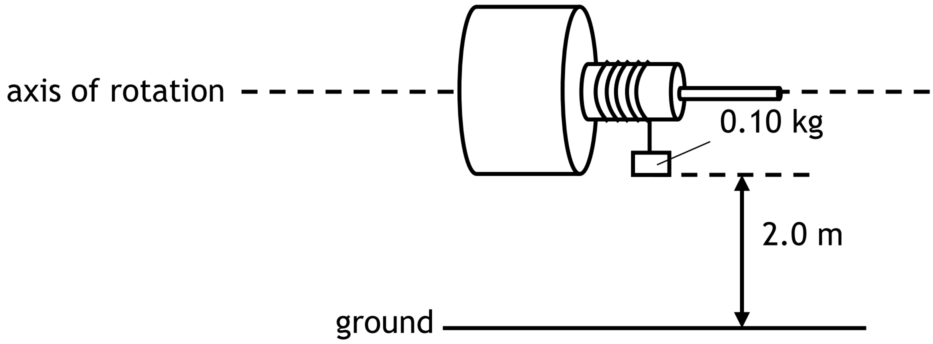
EXERCISE 5.2

Torque, Moments of Inertia and Angular Momentum

1. A flywheel has a moment of inertia of 1.2 kgm^2.
   It is acted on by a torque of magnitude 0.80 N m.

   (a) Calculate the angular acceleration produced.
   (b) The torque acts for 5.0 s and the flywheel starts from rest.
   Calculate the angular velocity at the end of the 5.0 s.

2. A mass of 0.10 kg is hung from the axle of a flywheel as shown below. The mass is released from a height of 2.0 m above ground level.



   The following results were obtained in the experiment:

time for mass to fall to the ground	**t = 8.0 s**
radius of axle	**R = 0.10 m**

   (a) By energy considerations, show that the final speed of the flywheel is given by:

$$v = \sqrt{\frac{3.92}{0.1+100I}}$$

   where I is the moment of inertia of the flywheel. Frictional effects are negligible.

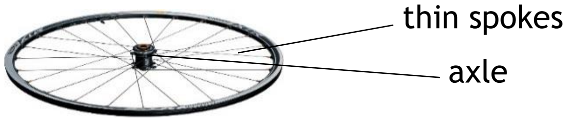
   (b) Calculate the moment of inertia of the flywheel.

3. A heavy drum has a moment of inertia of 2.0 kg m^2.
   It is rotating freely at 10 rev s^{-1} and has a radius of 0.50 m.
   A constant frictional force of 5.0 N is then exerted at the rim of the drum.

   (a) Calculate the time taken for the drum to come to rest.
   (b) Calculate the angular displacement in this time.
   (c) Hence calculate the heat generated in the braking action.

4. A cycle wheel is mounted so that it can rotate horizontally as shown.

Data on wheel: radius of wheel = 0.50 m mass of wheel = 2.0 kg



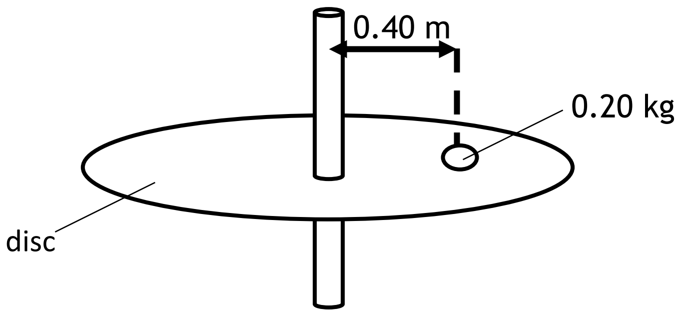
(a) Calculate the moment of inertia of the wheel system. State any assumptions you make.

(b) A constant driving force of 20 N is applied to the rim of the wheel.
Calculate the magnitude of the driving torque on the wheel.

(c) Calculate the angular acceleration of the wheel.

(d) After a period of 5.0 s, calculate:
(i) the angular displacement,
(ii) the angular momentum of the wheel
(iii) the kinetic energy of the wheel.

5. A very light but strong disc is mounted on a free turning bearing as shown below.



A mass of 0.20 kg is placed at a radius of 0.40 m and the disc is rotated at 1.0 revs^{-1}.
(The moment of inertia of the disc can be considered to be negligible.)

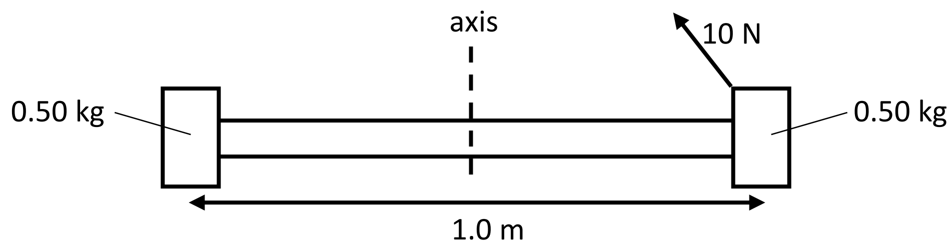
(a) Calculate the angular momentum of the 0.20 kg mass.

(b) Calculate the kinetic energy of the mass.

(c) The mass is pushed quickly into a radius of 0.20 m.
By applying the principle of conservation of angular momentum, calculate the new angular velocity of the mass in rads^{-1}.

(d) Calculate the new kinetic energy of the mass and account for any difference.

6. A uniform metal rod has a mass, M, of 1.2 kg and a length, L, of 1.0 m. Clamped to each end of the rod is a mass of 0.50 kg as shown below.



(a) Calculate an approximate value for the moment of inertia of the complete arrangement about the central axis as shown. Assume that $I_{rod} = \frac{1}{12}ML^2$ about this axis.

(b) The arrangement is set rotating by a force of 10 N as shown in the diagram. The force acts at a tangent to the radius.
Calculate the applied torque.

(c) Calculate the maximum angular acceleration.
You may assume that the force of friction is negligible.

(d) Calculate the kinetic energy of the arrangement 4.0 s after it is set rotating.

7. An unloaded flywheel, which has a moment of inertia of 1.5 kg m^2, is driven by an electric motor. The flywheel is rotating with a constant angular velocity of 52 rad s^{-1}. The driving torque, of 7.7 N m, supplied by the motor is now removed.

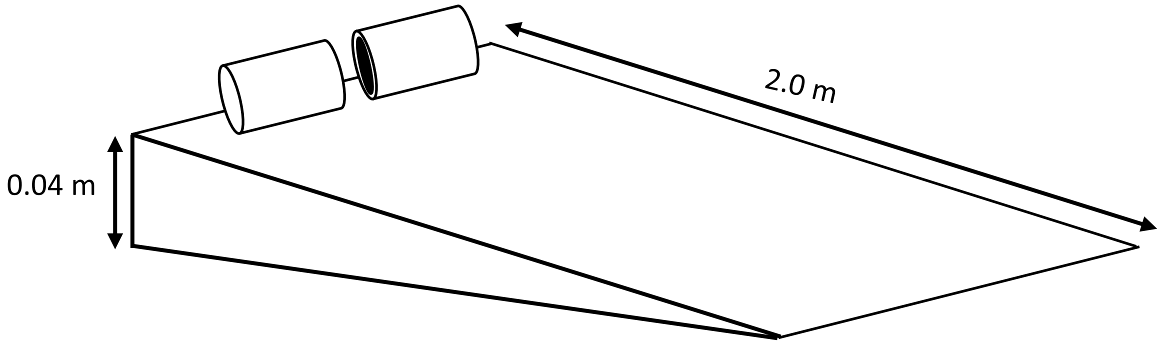
Calculate how long will it take for the flywheel to come to rest.
You may assume that the frictional torque remains constant.

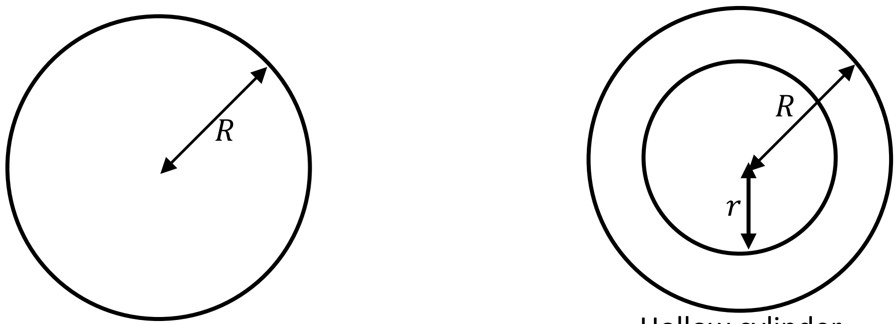
8. A solid aluminium cylinder and a hollow steel cylinder have the same mass and radius. The two cylinders are released together at the top of a slope.

(a) State which of the two cylinders will reach the bottom first.

(b) Explain your answer to part (a).

9. A solid cylinder and a hollow cylinder each having the same mass M and same outer radius R, are released at the same instant at the top of a slope 2.0 m long as shown below.
The height of the slope is 0.04 m.





Solid cylinder

$M = 10 \text{ kg} \quad R = 0.10 \text{ m}$

$$I = \frac{1}{2}MR^2$$

Hollow cylinder

$M = 10 \text{ kg} \quad R = 0.10 \text{ m} \quad r = 0.05 \text{ m}$

$$I = \frac{1}{2}M(R^2 + r^2)$$

It is observed that one of the cylinders reaches the bottom of the slope before the other.

(a) Using the expressions given above, show that the moments of inertia for the cylinders are as follows:
   (i) solid cylinder; $I = 0.05 \text{ kgm}^2$
   (ii) hollow cylinder; $I = 0.0625 \text{ kgm}^2$

(b) By energy considerations, show that the linear velocity of any cylinder at the bottom of the slope is given by:

$$v = \sqrt{\frac{2gh}{1 + \frac{I}{MR^2}}}$$

(c) Using the expression in (b) above, calculate the velocities of the two cylinders at the bottom of the slope and hence show that one of the cylinders arrives at the bottom of the slope 0.23 s ahead of the other.

EXERCISE 6.1

Gravitation

1. State the inverse square law of gravitation.

2. Calculate the gravitational force between two cars, each of mass 1000 kg, parked 0.50 m apart.

3. Calculate the gravitational force between the Earth and the Sun.

4.
   (a) By considering the force on a mass, at the surface of the Earth, state the expression for the gravitational field strength, g, in terms of the mass and radius of the Earth.
   (b) The gravitational field strength is 9.8 Nkg^{-1} at the surface of the Earth.
      (i) Calculate a value for the mass of the Earth.
      (ii) Calculate the gravitational field strength at the top of Ben Nevis, 1344 m above the surface of the Earth.
      (iii) Calculate the gravitational field strength at 200 km above the surface of the Earth.

5.
   (a) State what is meant by the gravitational potential at a point.
   (b) State the expression for the gravitational potential at a point.
   (c) Calculate the gravitational potential:
      (i) at the surface of the Earth
      (ii) 800 km above the surface of the Earth.

6. 'A gravitational field is a conservative field.' Explain what is meant by this statement.

7. Calculate the energy required to place a satellite of mass 200 000 kg into an orbit at a height of 350 km above the surface of the Earth.

8. Identify which of the following are vector quantities.

gravitational field strength	**gravitational potential**
escape velocity	**Universal Constant of Gravitation**
gravitational potential energy	**period of an orbit**

9. A mass of 8.0 kg is moved from a point in a gravitational field where the gravitational potential is –15 Jkg^{-1} to a point where the gravitational potential is –10 Jkg^{-1}.
   (a) Determine the potential difference between the two points.
   (b) Calculate the change in potential energy of the mass.
   (c) Calculate how much work would have to be done against gravity to move the mass between these two points.

10.
   (a) State what is meant by the term 'escape velocity'.
   (b) Derive an expression for the escape velocity in terms of the mass and radius of a planet.
   (c) Calculate the escape velocity from both
      (i) Earth
      (ii) the Moon.
   (d) Using your answers to (c) comment on the atmosphere of the Earth and the Moon.

11. Calculate the gravitational potential energy and the kinetic energy of a 2000 kg satellite in geostationary orbit above the Earth.

12. A central force required to keep a satellite in orbit.

    (a) Derive the expression for the orbital period in terms of the orbital radius.
    (b) A satellite is placed in a parking orbit above the equator of the Earth.
        (i) State the period of the orbit.
        (ii) Calculate the height of the satellite above the equator.
        (iii) Determine the linear speed of the satellite.
        (iv) Find the central acceleration of the satellite.

13. A white dwarf star has a radius of 8000 km and a mass of 1.2 solar masses.
    (a) Calculate the density of the star in kgm^{-3}.
    (b) Determine the gravitational potential at a point on the surface.
    (c) Calculate the acceleration due to gravity at a point on the surface.
    (d) Estimate the potential energy required to raise your centre of gravity from a sitting position to a standing position on this star.
    (e) A 2 kg mass is dropped from a height of 100 m on this star.
    Calculate how long it takes to reach the surface of the star.

14.
    (a) State how photons are affected by a massive object such as the Sun.
    (b) Explain, using a sketch, why light from a distant star passing close to the Sun may suggest that the star is at a different position from its ‘true’ position.
    (c) Explain what is meant by the term ‘black hole’.

EXERCISE 6.2

Gravitation

1. Show that the force of attraction between two large ships of mass 50,000 tonnes and separated by a distance of 20 m is 417 N. (1 tonne = 1000 kg)

2. Calculate the gravitational force of attraction between the proton and the electron in a hydrogen atom.
   Assume the electron is describing a circular orbit with a radius of 5.3×10^{-11} m.

   mass of proton = 1.673×10^{-27} kg **mass of electron = 9.11×10^{-31} kg**

3. A satellite, of mass 1500 kg, is moving at constant speed in a circular orbit 160 km above the Earth's surface.
   (a) Calculate the period of rotation of the satellite.
   (b) Calculate the total energy of the satellite in this orbit.
   (c) Calculate the minimum amount of extra energy required to boost this satellite into a geostationary orbit which is at a distance of 36 000 km above the Earth's surface.

4. The planet Mars has a mean radius of 3.4×10^6 m.
   The Earth's mean radius is 6.4×10^6 m.
   The mass of Mars is 0.11 times the mass of the Earth.
   (a) Determine how the mean density of Mars compares with that of the Earth.
   $\left(density, \rho = \frac{m}{V}\right)$
   (b) Calculate the value of "g" on the surface of Mars.
   (c) Calculate the escape velocity on Mars.

5. Determine the potential energy between the planet Saturn and its rings.
   The mass of Saturn is 5.72×10^{26} kg. The rings have a mass of 3.5×10^{18} kg and are concentrated at an average distance of 1.1×10^8 m from the centre of Saturn.

6. During trial firing of Pioneer Moon rockets, one rocket reached an altitude of 125 000 km. Neglecting the effect of the Moon, estimate the velocity with which this rocket struck the atmosphere of the Earth on its return. (Assume that the rocket's path is entirely radial and that the atmosphere extends to a height of 130 km above the Earth's surface).

7.
   (a) Sketch the gravitational field pattern between the Earth and Moon.
   (b) Gravity only exerts attractive forces. There should therefore be one position between the Earth and Moon where there is no gravitational field - a so-called 'null' point.
   By considering the forces acting on a mass m placed at this point, calculate how far this position is from the centre of the Earth.

8. Mars has two satellites named Phobos and Deimos. Phobos has an orbital radius of 9.4×10^6 m and an orbital period of 2.8×10^4 s.
   Using Kepler's third law, calculate the orbital period of Deimos (which has an orbital radius of 2.4×10^7 m).

9. When the Apollo 11 satellite took the first men to the Moon in 1969 its trajectory was very closely monitored.
   The satellite had a velocity of 5374 ms^{-1} when 26306 km from the centre of the Earth and this had dropped to 3560 m s-1 when it was 54368 km from the centre of the Earth. The rocket motors had not been used during this period.
   Calculate the gravitational potential difference between the two points. Remember that the unit of gravitational potential is Jkg^{-1}.

10. Show that an alternative expression for the escape velocity from a planet may be given by:

$$v_{esc} = \sqrt{2gR}$$

    where g = the planet’s surface gravitational attraction and R = the radius of the planet.

11. Show that a satellite orbiting the Earth at a height of 400 km has an orbital period of 93 minutes. Note that a height of 400 km is equal to a radius of R_E + 400 km.

12. A geostationary orbit has a period of approximately 24 hours.
    (a) Calculate the orbital radius for a satellite in such an orbit.
    (b) Determine the height of this satellite above the Earth.
    (c) Show on a sketch of the Earth the minimum number of geostationary satellites needed for world-wide communication.

EXERCISE 7.1

Space and time

1. State what is meant by an inertial frame of reference.

2. State what is meant by a non-inertial frame of reference.

3. State which frame of reference applies to the theory of special relativity (studied in Higher Physics).

4. State what range of speeds the results obtained by the theory of special relativity agree with those of Newtonian mechanics.

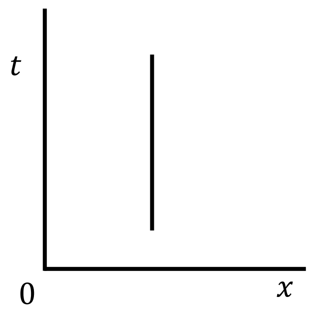
5. State how the equivalence principle link the effects of gravity with acceleration.

6. State which part of an accelerating spacecraft does time pass more slowly in.

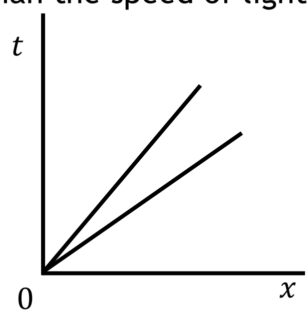
7. State how the passage of time is affected by moving to high altitudes in a gravitational field.

8. State how many dimensions are normally associated with space-time.

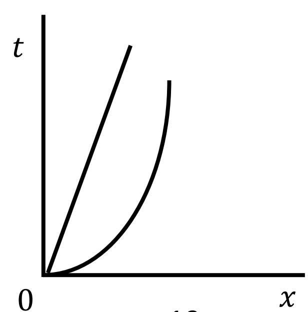
9. Write down what this worldline describes.



10. The space-time diagram shows two worldlines. State which worldline describes a faster speed
(These speeds are much less than the speed of light.)

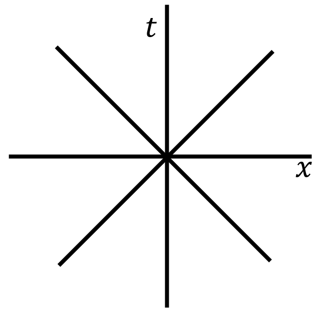


11. Explain the difference between these two worldlines on the space-time diagram.



12. Explain what is meant by the term geodesic.

13. Copy the following space-time diagram.



Label your diagram with the following terms:

the present **the future** **the past** **v = c** **v < c** **v > c**

14. State the effect mass has on spacetime.

15. Describe two situations where a human body experiences the sensation of force.

16. Describe how general relativity interprets the cause of gravity.

17. A star of mass 4.5×10^{31} kg collapses to form a black hole.
Calculate the Schwarzschild radius of this black hole.

18. A star of mass equivalent to six solar masses collapses to form a black hole.
Calculate the Schwarzschild radius of this black hole.

19. Calculate the Schwarzschild radius of the black hole which would form if our Sun collapsed.

20. Calculate the Schwarzschild radius of the black hole which would form if our Earth collapsed.

21. A star is the same size as our Sun and has an average density of 2.2×10^3 kg m^{-3}.
If this star collapsed to form a black hole, calculate the Schwarzschild radius of the black hole.

$$\left(density, \rho = \frac{m}{V}\right)$$

22. Calculate the mass of a black hole with a Schwarzschild radius of 150 m.

EXERCISE 8.1

Stellar physics

1. A star emits electromagnetic radiation with a peak wavelength of 6.8×10^{-7} m.
   (a) Use Wien's law ($\lambda_{max}T = 2.9 \times 10^{-3}$) to calculate the surface temperature of the star.
   (b) Calculate the power of the radiation emitted by each square metre of the star's surface where the star is assumed to be a black body.
   Stefan-Boltzmann constant = 5.67×10^{-8} $Js^{-1}m^{-2}K^{-4}$.

2. The Sun has a radius of 7.0×10^{8} m and a surface temperature of 5800 K.
   (a) Calculate the power emitted per m^2 from the Sun's surface.
   (b) Calculate the luminosity of the Sun.
   (c) Calculate the apparent brightness of the Sun as seen from the Earth.

3. Three measurements of a distant star are possible from Earth.
   These measurements are:

   apparent brightness = 4.3×10^{-9} Wm^{-2}
   peak emitted wavelength = 2.4×10^{-7} m
   distance to star (parallax method) = 8.5×10^{17} m

   (a) Use Wien's law ($\lambda_{max}T = 3 \times 10^{-3}$) to calculate the surface temperature of the star.
   (b) Calculate the energy emitted by each square metre of the star's surface per second.
   (c) Calculate the luminosity of the star.
   (d) Calculate the radius of the star.

4. A star is 86 light years from Earth and has a luminosity of 4.8×10^{28} W.
   Calculate the apparent brightness of the star.

5. The apparent brightness of a star is 6.2×10^{-8} Wm^{-2}. The star is 16 light years from Earth.
   Calculate the luminosity of the star.

6. A star with luminosity 2.1×10^{30} W has an apparent brightness of 7.9×10^{-8} Wm^{-2} when viewed from Earth.
   Calculate the distance of the star from Earth:
   (a) in metres
   (b) in light years.

7. A star with a radius of 7.8×10^{8} m and a surface temperature of 6300 K has an apparent brightness of 1.8×10^{-8} Wm^{-2}.
   Calculate its distance from the Earth.

8. A star with radius 9.5×10^{9} m and surface temperature 5900 K is 36 light years from Earth.
   Calculate the apparent brightness of the star.

9. Show mathematically that the luminosity of a star varies directly with the square of its radius and the fourth power of its surface temperature.

10. Show mathematically that the apparent brightness of a star varies directly with the square of its radius and the fourth power of its surface temperature and varies inversely with the square of its distance from the Earth.

11. Two stars, A and B, are the same distance from the Earth.
The apparent brightness of star A is 8.0×10^{-12} Wm^{-2} and the apparent brightness of star B is 4.0×10^{-13} Wm^{-2}.
Show that star A has 20 times the luminosity of star B.

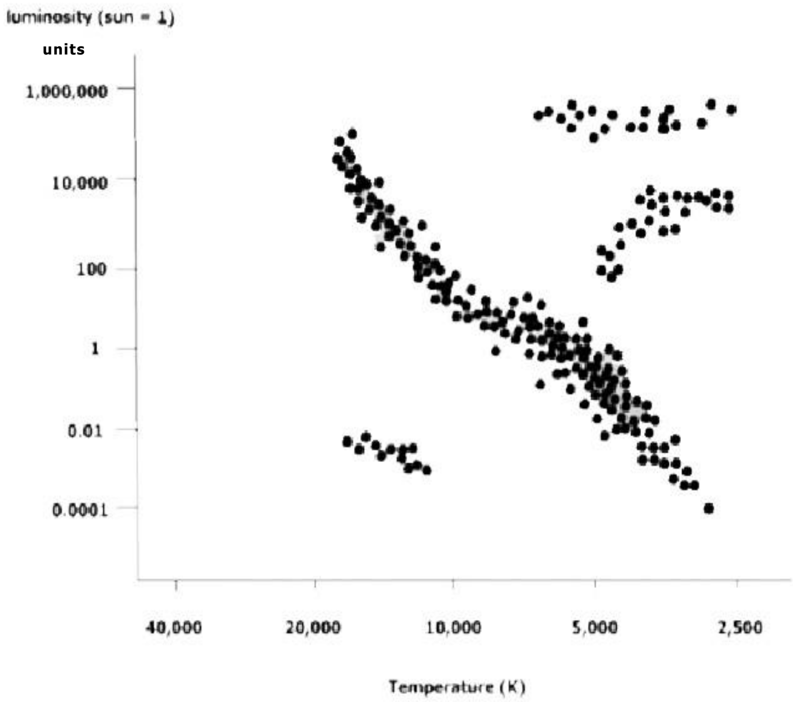
12. A star has half of our Sun's surface temperature and 400 times our Sun's luminosity.
Calculate how many times bigger is the radius of this star compared to the Sun.

13. Information about two stars A and B is given below.

surface temperature of star:	**A = 3 × surface temperature of star B**
radius of star:	**A = 2 × radius of star B**

(a) How many times is the luminosity of star A greater than the luminosity of star B?
(b) Stars A and B have the same apparent brightness from Earth.
Identify which star is furthest from Earth, and calculate how many times further away it is.

14. The diagram shows one way of classifying stars. Each dot on the diagram represents a star.



(a) State the name given to this type of diagram.
(b) The stars are arranged into four main regions. Identify the region called:
(i) the main sequence
(ii) giants
(iii) super giants
(iv) white dwarfs.
(c)
(i) State the region in which we would find our Sun.
(ii) The surface temperature of the Sun is approximately 5800 K.
Explain why the scale on the temperature axis makes it difficult to identify which dot represents the Sun.
(d) Identify the region in which the following are found:
(i) a hot bright star
(ii) a hot dim star
(iii) a cool bright star
(iv) a cool dim star
(e) A star is cooler than, but brighter than the Sun.
(i) Determine how the size of this star compares to the size of our Sun.
(ii) Identify what region would this star be in.
(f) A star is hotter than, but dimmer than, the Sun.
(i) Determine how the size of this star compares to the size of our Sun.
(ii) Identify what region would this star be in.
(g) The Sun's nuclear fuel will be used up with time. State what will then happen to the Sun's position in the above diagram.